MAT 2377, Probability and statistics for engineers

Assignment 1 - solutions

[2] **Exercise 2-36:** By the multiplication principle, there are $3 \cdot 4 \cdot 3 = 36$ different ways to choose the three tools.

[6] **Exercise 2-44:**

(a) The permutation of the 12 components gives us the design. Thus, there are $P_{12}^{12} = 12! = 479\,001\,600$ different designs.

(b) To obtain one of the designs, we can select 7 positions from the 12 for the 7 identical components and then order the other components to determine their positions among the 5 remaining positions. So there are

$$\left(\begin{array}{c} 12 \\ 7 \end{array} \right) \cdot P_5^5 = (792)\,(120) = 95\,040 \text{ different designs.}$$

(c) To obtain one of the designs, we can select 3 positions among the 12 for the 3 identical components. The next step would be to choose 4 positions among the 9 remaining positions for the 4 identical components. To finish, we will order the 5 remaining components to determine their positions. There are

$$\left(\begin{array}{c} 12 \\ 3 \end{array} \right) \left(\begin{array}{c} 9 \\ 4 \end{array} \right) \cdot P_5^5 = (220)(126)\,(120) = 3\,326\,400 \text{ possible designs.}$$

[4] **Exercise 2-48:**

(a) The number of samples with exactly one non-conforming part is

$$\left(\begin{array}{c} 2 \\ 1 \end{array} \right) \left(\begin{array}{c} 10 \\ 2 \end{array} \right) = (2)(45) = 90.$$

(b) The number of samples that contain at least one non-conforming part is the difference between the total number of samples and the number of samples with no non-conforming parts:

$$\left(\begin{array}{c} 12 \\ 3 \end{array} \right) - \left(\begin{array}{c} 10 \\ 3 \end{array} \right) = 220 - 120 = 100.$$

[6] **Exercise 2-82:**

(a) $P(A') = 1 - P(A) = 0.7$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.1 = 0.4$

(c) $P(A' \cap B) = P(B) - P(A \cap B) = 0.2 - 0.1 = 0.1$

(d) $P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.1 = 0.2$

(e) $P[(A \cup B)'] = 1 - P(A \cup B) = 1 - 0.4 = 0.6$

(f) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B) = 0.7 + 0.2 - 0.1 = 0.8$

[4] **Exercise 2-92:** We have $P(A) = 1/4$, since red of one of 4 colours; $P(B) = 4/5$, since 4 of the 5 font sizes are not the smallest. Furthermore, by the multiplication principle, by considering the choice of the colour and the size of the font:

$$P(A \cap B) = \frac{1 \cdot 2}{4 \cdot 5} = \frac{1}{10}.$$

(a) $P(A' \cap B) = P(B) - P(A \cap B) = 0.7$

(b) $P(A \cap B') = P(A) - P(A \cap B) = 0.12$

(c) $P[(A \cup B)'] = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] = 0.05$

[2] **Exercise 2-122:** By the total probability rule, we have

$$P(A) = P(A|B)P(B) + P(A|B')P(B') = (0.2)(0.8) + (0.3)(1 - 0.8) = 0.22.$$

[4] **Exercise 2-180:** Let T be the event that the order has a tent and let M be the event that the order has a mattress. We have

$$P(M|T) = 0.4, P(M|T') = 0.05, P(T) = 0.2.$$

(a) The probability that an order includes a mattress is

$$\begin{aligned} P(M) &= P(M|T)P(T) + P(M|T')P(T') \\ &= (0.4)(0.2) + (0.05)(1 - 0.2) = 0.12. \end{aligned}$$

(b) The probability that the order includes a tent, given that it includes a mattress, is

$$P(T|M) = \frac{P(T \cap M)}{P(M)} = \frac{P(M|T)P(T))}{P(M)} = \frac{(0.4)(0.2)}{0.12} = 0.66667.$$

[4] **Exercise 2-220:** Let E, A_s, A_o and A_p be the events that there is a read error, that there is a skewed alignment, that the alignment is off-center and that there is a proper alignment, respectively. We are given:

$$P(A_s) = 0.1, P(A_o) = 0.05, P(A_p) = 0.85,$$

and

$$P(E|A_s) = 0.01, P(E|A_o) = 0.02, P(E|A_p) = 0.001.$$

(a) By the total probability rule, we get

$$\begin{aligned} P(E) &= P(E|A_s)P(A_s) + P(E|A_o)P(A_o) + P(E|A_p)P(A_p) \\ &= (0.01)(0.1) + (0.02)(0.05) + (0.001)(0.85) = 0.00285. \end{aligned}$$

(b) We want

$$P(A_s|E) = \frac{P(A_s \cap E)}{P(E)} = \frac{P(E|A_s)P(A_s))}{P(E)} = \frac{(0.01)(0.1)}{0.00285} = 0.3509.$$

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